Minimum MSE Criteria Design of Amplify-and-Forward MIMO Relay Wireless Communication System

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Abstract: In this paper, we investigate a channel estimation scheme for a two-hop multiple input multiple output (MIMO) relay system. The amplify-and-forward (AF) relaying is an efficient way to increase radio range and improve link reliability with a low implementation cost. Furthermore, the research about AF relay in a single antenna scenario has matured. Why the applications can using for broadband radio transmission using MIMO techniques still require more effort to overcome some practical challenges, such as significant overhead for channel estimation and reliability for channel estimation errors. This paper reviews some recently proposed linear transceiver designs based on the minimum mean squared error (MSE) criterion and shows that the proposed MSE decomposition and relaxation method can reach a suitable solution for those challenges. Numerical results reveal the effectiveness of the proposed schemes.

Keywords: multiple-input multiple-output, amplify-and-forward, relay, minimum mean squared error

Introduction

Multiple Input Multiple Output (MIMO) relay wireless communication that incorporates relay technology and MIMO wireless network. It has attracted considerable attention in recent years due to its potential ability to extend network coverage and improve link reliability as well as spectral efficiency [2] - [11]. This paper aims to provide an overview of optimal source design of amplify-and-forward MIMO relay networks using the minimum MSE criteria.

A MIMO relay can be regenerative or non-regenerative, full-duplex or half-duplex, one-way, or two way [2]-[10]. A regenerative relay requires digital decoding and re-encoding at the relay, which can cause a significant increase of delay and complexity. A non-regenerative relay does not need any digital decoding and re-encoding at the relay, which is a useful advantage over regenerative relays. The separation between the full-duplex relay and half-duplex relay depends on whether relays can transmit and receive at the same time or same frequency or not. Full-duplex relay is spectrally practical but induces a problem of self-interference, and the half-duplex relay is easy to achieve but not spectrally efficient as full-duplex. The difference between two-way relay and the one-way relay is whether relays can relay information in two directions in a single time or single frequency or not. In this paper, we will concentrate on non-regenerative, half-duplex, one-way MIMO relay systems. And then the fundamental motivation behind the use of cooperative communications extends in the exploitation of spatial diversity provided by the network nodes [5] - [8], as well as the efficient use of power resources [9]-[10] which can be achieved by a scheme that quickly receives and forwards a given information, yet designed under particular optimality criterion.

The relay scheme could be categorized into three general groups: amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF). In the AF scheme, the relay nodes amplify the received signal and rebroadcast the amplified signals toward the destination node [5]-[9]. In the DF scheme, the relay nodes first decode the received signals and then forward the re-encoded signals toward the destination node [3]. In the CF method, the relay nodes compress the received signals by exploiting the statistical dependencies between the signals at the nodes [4]. In this paper, we consider the AF strategy, which is more straightforward to implement compared with the other two approaches.

Initial research on MIMO relay began in single user [2], [6] and [11], where the capacity bounds of MIMO relay systems were examined. It has been shown in [7] and [9] that performing linear processing at the relay node can outperform the conventional AF relaying in a non-regenerative, also known as amplify-and-forward (AF), MIMO relay system. The optimal relay-amplifying matrix which, maximizes the mutual information (MI) between source and destination is derived in [10]. In [8], a minimum mean square error (MMSE) based iterative algorithm is proposed for jointly designing the source, relay and destination processing matrices. A unified framework is developed in [11] to jointly optimize the source preceding matrix and relay amplifying matrix for a broad class of objective functions. The application of the design can be applied, such as at the navigation based on the laser sensor is derived in [12].

In this paper, we focus on optimization for single user non-regenerative MIMO relay wireless systems where each node is equipped with multiple antennas supporting multiple data streams. We explore of minimum
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mean squared error (MMSE) algorithms to optimal source preceding matrix. Note that the MMSE algorithms have already been studied with single-hop MIMO [1] and MIMO relay channels [2] and [6]. Our results show that the MMSE algorithm for MIMO relay system has significant performance improvement over the system without un-optimal the source matrix.

The rest of the paper is organized as follows with the system model is described in Section II. In Section III, we study the MMSE algorithm of a MIMO relay system. And then, section IV shows the simulation results which justify the performance gain with optimal source matrix the MMSE algorithms under various system scenarios. Finally, the conclusion is given in section V.

System Model

Our proposed optimal source design MIMO relay system is illustrated in Fig. 2. For simplicity, we consider a three-node MIMO communication system where the source node transmits information to the destination node with the aid of one relay node. The source, the relay and the destination nodes are equipped with $N_s$, $N_r$, and $N_d$ antennas, respectively. All data streams were transmitted through the relay, and there is no direct link between the source and the destination. $H_{s,r}$ and $H_{d}$ are the channel matrices for the source-relay and the relay-destination links with dimensions $N_r \times N_s$ and $N_d \times N_r$, respectively. The communication process between the source and destination nodes is completed in two-time slots.

In the first time slot, the modulated symbol $s$ is linearly pre-coded as

$$x = bs \tag{1}$$

Where $b$ is an $N_s \times 1$ transmit beamforming vector. The pre-coded vector $x$ is then transmitted to the relay nodes. The received signal at the relay node can be written as

$$y_r = H_{s,r}x + v_r \tag{2}$$

Where $y_r$ and $v_r$ are the received signal and the additive Gaussian noise vectors at the relay node, respectively. In the second slot, the source node remains silent, and the relay node multiplies (linearly pre-codes) the received signal vector by an $N_r \times N_r$ relay amplifying matrix $F$ and transmits the pre-coded signal vector $x_r$ to the destination node. Thus

$$x_r = Fy_r \tag{3}$$

The received signal vector at the destination node can be written as

$$y_d = H_{s}x_r + v_d
= H_{s}Fy_r + v_d
= H_{s}F(H_{s,r}s + v_r) + v_d
= H_{s}FH_{s,r}s + H_{s}Fv_r + v_d \tag{4}$$

Where $y_d$ and $v_d$ are the received signal and additive Gaussian noise vectors at the destination node, respectively. Thus we see from the above equation that the received signal vector at the destination can be equivalently written as

$$y_d = Hs + v \tag{5}$$

where $H = H_{s}FH_{s,r}$ is the equivalent MIMO channel and $v = H_{s}Fv_r + v_d$ is the equivalent noise.

In this paper, we try to improve the system BER performance by using linear receiver MMSE. A simple approach to design the relay is to treat it as an all-pass AF unit, which we construct as $F = aI_{N_r}$, where $a$ is the amplifying factor of the relay and $I_{N_r}$ is an identity matrix of dimension $N_r$. We can find from $P_r = a^2tr\{P_s/N_rH_{s,r}H_{s,r}^H + I_{N_r}\}$, Here $P_s > 0$ and $P_r > 0$ are the transmit power available at the source and the relay nodes respectively, $(\cdot)^H$ denotes matrix Hermitian and $tr\{\cdot\}$ indicates the trace of a matrix.
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**Figure 1:** MIMO Relay Wireless Communication Network

**Figure 2:** Block diagram of the equivalent MIMO Relay system

### Linear Receiver MMSE MIMO Relay System

In this section, we develop the optimal transmit beamforming vector $b$ and the relay amplifying matrix $F$ to minimize the MSE of the signal waveform estimation. By using a linear receiver, the estimated signal waveform vector at the destination node is given by

$$s_{new} = W^H y_d$$  \(6\)

Where $W$ is an $N_d \times N_b$ weight matrix, and $(\cdot)^H$ denotes the matrix (vector) Hermitian transpose. The minimum MSE (MMSE) approach tries to find a weight matrix $W$ that minimizes the statistical expectation of the signal waveform estimation given by

$$\text{MSE} = \text{tr}(E[(s - \hat{s})(s - \hat{s})^H])$$  \(7\)

Where $\text{tr} (\cdot)$ stands for the matrix trace, and $E[\cdot]$ denotes statistical expectation. We assume that the source signal vector satisfies $E[s b^H] = I_{N_b}$, where $I_n$ is an $n \times n$ identity matrix, and all noises are independent and identically distributed with zero mean and unit variance. Substituting (6) into (7), we find that the $W$ that minimizes (7) by using the equation from [2], then it can be written as

$$W = (\bar{H}H^H + C)^{-1}H$$  \(8\)

Where $(\cdot)^{-1}$ denotes the matrix inversion, $\bar{H}$ and $F$ are from [2], and the equivalent noise covariance matrix written as

$$C = FF^H + I_{N_d}$$

By using (8), it can be seen that the MSE is a function of $\bar{F}$ and can be written as

$$\text{MSE} = \text{tr}((I_{N_b} + \bar{R}^H \tilde{C}^{-1}\bar{R})^{-1})$$

By following algorithm for MIMO relay systems using linear receiver MMSE. It consider to the received signal vector at the destination in (5), then our proposed MIMO relay channel (Fig. 2). Then, It will reduce to a MIMO channel with the equivalent channel matrix of $H = H_{s}F_{s}H_{s}d$, the signals vector of $s$ and then the equivalent
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noise vector of $\mathbf{v} = \mathbf{H}_r \mathbf{F} \mathbf{v}_r + \mathbf{v}_d$.

Simulation Results and Discussions

In the simulations, the transmission signalling is in spatial multiplexing mode (i.e., the source transmits independent data streams from different antennas) with total transmit power uniformly distributed among the transmit antennas. Also, all simulations are conducted in a flat-fading Raleigh environment using the BPSK constellation, and the noise variances are assumed to be the same for all antennas. We transmitted 100 randomly generated bits in each channel realization, and the BER results are averaged through 200 channel realizations. We plot BER curves versus SNRs.

In the first example, we simulate the system BER performance of MMSE receiver compared with the zero-forcing algorithm [2] of MIMO relay channel with varying SNR in the source to relay link ($\text{SNR}_s$) keeping the relay to destination SNR ($\text{SNR}_r$) at 25 dB. Fig. 4 show the BER performance with $N_s = N_r = N_d = 4$, where Fig. 3 show four antennas at source note, four antennas at the relay node and four antennas at the destination node. It can be seen that, at $\text{BER} = 10^{-2}$, we achieve a 5 dB gain from the [2] to the proposed algorithm.

![Fig. 3. Block diagram of MIMO Relay with four antennas at the source, relay and destination.](image)

![Fig. 4: BER versus $\text{SNR}_s$. $N_s = N_r = N_d = 4$ and $\text{SNR}_r = 25$ dB for MIMO relay channel.](image)
In the second example, we simulate the system BER performance of MMSE receiver MIMO relay channel with varying SNR in the $SNR_s$ keeping the $SNR_r$ at 25 dB. Fig. 5 show the BER performance with varying of $N_s = 4$, 6, and 8 with $N_t = N_d = 4$. Our results demonstrate that MMSE MIMO Relay algorithm achieves by increasing the number of antennas.

![Fig. 5: BER versus $SNR_s$. $N_s = N_d = 4$, $N_r = 4$; $6$; $8$ and $SNR_r = 25$ dB for MIMO relay channel.](image)

In the third example, we simulate the system BER performance of MMSE receiver MIMO relay channel with varying SNR in the $SNR_s$ keeping the $SNR_r$ at 30 dB. Fig. 6 show the BER performance with varying $N_s = 4$, 6, and 8 with $N_t = N_d = 4$. Our results demonstrate that MMSE MIMO Relay algorithm achieves by increasing the number of antennas while we consider with $SNR_r$ at 30 dB.

![Fig. 6: BER versus $SNR_s$. $N_s = N_d = 4$; $N_r = 4$, 6, 8; and $SNR_r = 30$ dB for MIMO relay channel.](image)

**Conclusions**

In conclusion, we have demonstrated the advantage of using linear receiver MMSE MIMO relay wireless communication system. We designed relays as all-pass amplify-and-forward (AF) units that are simpler to implement. Our results demonstrate that MMSE MIMO relay algorithms outperform the zero-forcing
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algorithm.

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